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ON THE PSEUDO-LINEARIZATION PROBLEM FOR NONLINEAR  
SYSTEMS(U) JOHNS HOPKINS UNIV BALTIMORE MD DEPT OF  
ELECTRICAL AND COMPUT... J WANG ET AL. 01 MAR 88

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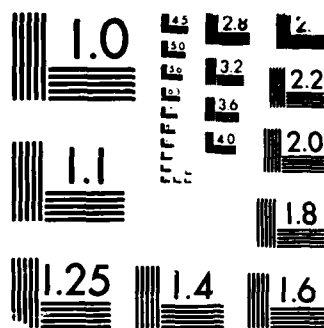
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## REPORT DOCUMENTATION PAGE

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Unclassified			1b. RESTRICTIVE MARKINGS	
2a. SECURITY CLASSIFICATION AUTHORITY			3. DISTRIBUTION / AVAILABILITY OF REPORT Approved for Public Release; Distribution unlimited	
2b. DECLASSIFICATION / DOWNGRADING SCHEDULE MAY 03 1988			5. MONITORING ORGANIZATION REPORT NUMBER(S) AFOSR-TR-88-0438	
4. PERFORMING ORGANIZATION REPORT NUMBER(S)				
6a. NAME OF PERFORMING ORGANIZATION Johns Hopkins University		6b. OFFICE SYMBOL (If applicable)		7a. NAME OF MONITORING ORGANIZATION AFOSR/NM
6c. ADDRESS (City, State, and ZIP Code) Baltimore, Maryland 21218		7b. ADDRESS (City, State, and ZIP Code) Bldg. 410 Bolling AFB, DC 20332-6448		
8a. NAME OF FUNDING / SPONSORING ORGANIZATION AFOSR		8b. OFFICE SYMBOL (If applicable) NM		9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER AFOSR-87-0101
8c. ADDRESS (City, State, and ZIP Code) Bldg. 410 Bolling AFB, DC 20332-6448		10. SOURCE OF FUNDING NUMBERS		
		PROGRAM ELEMENT NO. 61102F	PROJECT NO. 2304	TASK NO. A1
		WORK UNIT ACCESSION NO.		
11. TITLE (Include Security Classification) On the Pseudo-Linearization Problem for Nonlinear Systems				
12. PERSONAL AUTHOR(S) Rugh, Wilson J.				
13a. TYPE OF REPORT Journal Reprint		13b. TIME COVERED FROM 3/1/87 TO 2/29/88		14. DATE OF REPORT (Year, Month, Day) 1988, March 1
15. PAGE COUNT 45				
16. SUPPLEMENTARY NOTATION				
17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)	
FIELD	GROUP	SUB-GROUP		
			Nonlinear Control Systems; Control Theory	
19. ABSTRACT (Continue on reverse if necessary and identify by block number)				
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20. DISTRIBUTION / AVAILABILITY OF ABSTRACT <input checked="" type="checkbox"/> UNCLASSIFIED/UNLIMITED <input type="checkbox"/> SAME AS RPT <input type="checkbox"/> DTIC USERS			21. ABSTRACT SECURITY CLASSIFICATION Unclassified	
22a. NAME OF RESPONSIBLE INDIVIDUAL Maj. James M. Crowley			22b. TELEPHONE (Include Area Code) (202) 767-5025	22c. OFFICE SYMBOL NM

**On the Pseudo-Linearization Problem  
for Nonlinear Systems**

*Jianliang Wang and Wilson J. Rugh*

Department of Electrical and Computer Engineering  
The Johns Hopkins University  
Baltimore, Maryland 21218  
(301)338-7004

Technical Report JHU/ECE 87-18

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*Jianliang Wang and Wilson J. Rugh*

Department of Electrical and Computer Engineering  
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### ABSTRACT

Based on well-known results for linear systems, an alternate treatment of the pseudo-linearization problem of Reboulet and Champetier is given. Sufficient conditions for pseudo-linearization are obtained in the general case, and these are shown to be satisfied under mild hypotheses by systems with one or two inputs. Advantages of the new approach include the simplicity of the derivation, and a more explicit representation for a pseudo-linearizing transformation.



### Introduction

Pseudo-linearization of a nonlinear system involves computing a state feedback law and state variable change such that the closed-loop system described in terms of the new variables has a family of linearizations that is independent of the closed-loop constant operating point. This notion was introduced by Reboulet and Champetier, and has been treated from a differential-geometric viewpoint in a sequence of recent papers, [1-3]. We describe here an approach to pseudo-linearization based on the linearization family of the nonlinear system, using a linear transformation due to Luenberger [4], a linear feedback law due to Ackermann

Research sponsored by the Air Force Office of Scientific Research, Air Force Systems Command, USAF, under Grant Number AFOSR-87-0101.

[5], and results on linearization families in [6, 7]. This leads to an alternate derivation of the results in [1, 2] for one- and two-input systems, though in the general multi-input case only a sufficient condition is obtained. In the course of the derivation, explicit representations are obtained for a feedback law and a variable change that accomplish pseudo-linearization.

We adopt a local viewpoint, though when particular examples are considered the problem often can be solved in a nonlocal fashion. For simplicity, all functions and their derivatives that appear in the sequel are assumed to be continuous.

Multi-input nonlinear systems of the form

$$\dot{x}(t) = f(x(t), u(t)), \quad x(t) \in R^n, \quad u(t) \in R^m, \quad t \geq 0 \quad (1)$$

will be considered. It is assumed that (1) has a constant operating point family  $\{[x(\alpha), u(\alpha)], \alpha \in \Gamma\}$ , where  $\Gamma$  is an open neighborhood of  $0 \in R^m$  and, for convenience,  $x(0) = 0, u(0) = 0$ . That is, a constant input  $u(t) = u(\alpha_1)$  for all  $t \geq 0$ , where  $\alpha_1 \in \Gamma$  is fixed, and initial condition  $x(\alpha_1)$  yield the constant response  $x(t) = x(\alpha_1)$ . Typically the constant operating point family is parameterized by constant values of the input components and/or state components. We implicitly permit the shrinking of  $\Gamma$  in order to economically state local results.

Linearizing (1) about its constant operating point family yields the parameterized linearization family

$$\frac{d}{dt}[x(t) - x(\alpha)] = F(\alpha)[x(t) - x(\alpha)] + G(\alpha)[u(t) - u(\alpha)], \quad \alpha \in \Gamma \quad (2)$$

It will be assumed that for each  $\alpha \in \Gamma$ ,

$$\begin{aligned} \text{rank } G(\alpha) &= m, \quad \text{rank } \frac{\partial x}{\partial \alpha}(\alpha) = m \\ \text{rank } [G(\alpha) \ F(\alpha)G(\alpha) \ \cdots \ F^{n-1}(\alpha)G(\alpha)] &= n \end{aligned} \quad (3)$$

Finally we assume that the controllability indices, denoted by  $k_1, \dots, k_m$ , are constant for all  $\alpha \in \Gamma$ .

It should become clear that the assumptions of controllability and constant controllability indices are required for the pseudo-linearization problem to be well posed. The rank assumption on the operating point state is automatically satisfied if the constant operating point family is parameterized by constant values of state components, and in any case is

needed to insure that knowledge of  $x(\alpha)$  determines  $\alpha$ . Also, for use in the sequel, we note that [6]

$$F(\alpha) \frac{\partial x}{\partial \alpha}(\alpha) + G(\alpha) \frac{\partial u}{\partial \alpha}(\alpha) = 0, \text{ for all } \alpha \in \Gamma \quad (4)$$

### Pseudo-Linearization

The pseudo-linearization problem for (1) involves finding an invertible state variable change  $z(t) = P(x(t))$  and a feedback law  $u(t) = P_{n+1}(x(t), w(t))$ ,  $w(t) \in R^m$ , with  $[\partial P_{n+1}/\partial w](0, 0)$  nonsingular, such that the closed-loop system shown in Figure 1, when linearized about its constant operating point family  $\{[w(\alpha), z(\alpha)], \alpha \in \Gamma\}$ , has a constant-parameter linearization family described in the Brunovsky canonical form. That is, the closed-loop linearization family is required to take the form

$$\frac{d}{dt}[z(t) - z(\alpha)] = \bar{F}[z(t) - z(\alpha)] + \bar{G}[w(t) - w(\alpha)], \alpha \in \Gamma \quad (5)$$

where

$$\begin{aligned} \bar{F} &= \text{block diagonal } \{\bar{F}_1, \dots, \bar{F}_m\}, \quad \bar{F}_j = \begin{bmatrix} 0 & 1 & & 0 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & & 0 & 1 \\ 0 & 0 & & 0 & 0 \end{bmatrix} \in R^{k_j \times k_j} \\ \bar{G} &= \text{block diagonal } \{\bar{g}_1, \dots, \bar{g}_m\}, \quad \bar{g}_j = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \in R^{k_j \times 1} \end{aligned} \quad (6)$$

More or less implicit in this problem statement is the need to determine  $w(\alpha)$  and  $z(\alpha)$  in a manner consistent with the variable change and the requirement that (5) be a linearization family for the closed-loop nonlinear system. Specifically, the closed-loop constant operating point family must satisfy

$$z(\alpha) = P(x(\alpha)), \quad u(\alpha) = P_{n+1}(x(\alpha), w(\alpha)), \quad \alpha \in \Gamma \quad (7)$$

and, for convenience, we will require  $P(0) = 0$  and  $P_{n+1}(0, 0) = 0$ . Also, by the condition analogous to (4) applied to the closed-loop linearization (5) we must have, for all  $\alpha \in \Gamma$ ,

$$w(\alpha) = 0, \quad z_i(\alpha) = 0, \quad i = \begin{cases} 2, \dots, k_1, \\ k_1+2, \dots, k_2, \\ \dots, \\ k_1 + \dots + k_{m-1} + 2, \dots, n \end{cases} \quad (8)$$

Differentiating the first condition in (7) and combining with (8) gives the requirement (writing transposes to save space)

$$\begin{aligned} & \left[ \left[ \frac{\partial z_1}{\partial \alpha}(\alpha) \right]^T 0 \dots 0 \left[ \frac{\partial z_{k_1+1}}{\partial \alpha}(\alpha) \right]^T 0 \dots 0 \dots \left[ \frac{\partial z_{k_1 + \dots + k_{m-1} + 1}}{\partial \alpha}(\alpha) \right]^T 0 \dots 0 \right]^T \\ & = \frac{\partial P}{\partial x}(x(\alpha)) \frac{\partial x}{\partial \alpha}(\alpha) \end{aligned} \quad (9)$$

To solve the pseudo-linearization problem, we first show how to choose  $K(\alpha) \in R^{m \times m}$ , invertible  $M(\alpha) \in R^{m \times m}$ , and invertible  $Q(\alpha) \in R^{n \times m}$  such that, for all  $\alpha \in \Gamma$ ,

$$Q(\alpha)[F(\alpha) + G(\alpha)K(\alpha)]Q^{-1}(\alpha) = \bar{F}, \quad Q(\alpha)G(\alpha)M(\alpha) = \bar{G} \quad (10)$$

Secondly, we will show how to compute  $P_{n+1}(x, w)$  such that, for all  $\alpha \in \Gamma$ ,

$$\begin{aligned} P_{n+1}(x(\alpha), w(\alpha)) &= u(\alpha), \quad \frac{\partial P_{n+1}}{\partial x}(x(\alpha), w(\alpha)) = K(\alpha), \\ \frac{\partial P_{n+1}}{\partial w}(x(\alpha), w(\alpha)) &= M(\alpha) \end{aligned} \quad (11)$$

where  $w(\alpha) \equiv 0$ . Finally, we will determine conditions under which (9) can be satisfied with  $[\partial P / \partial x](x(\alpha)) = Q(\alpha)$ , and show how to compute  $P(x)$  such that, for all  $\alpha \in \Gamma$ ,

$$P(x(\alpha)) = z(\alpha), \quad \frac{\partial P}{\partial x}(x(\alpha)) = Q(\alpha) \quad (12)$$

In order to specify in an explicit fashion the matrices  $Q(\alpha)$ ,  $K(\alpha)$  and  $M(\alpha)$  that satisfy (10), we will make use of developments leading to the Luenberger controller form [4] and the so-called minimum-time deadbeat control law for discrete-time systems due to Ackermann [5]. Denoting the columns of  $G(\alpha)$  by  $g_1(\alpha), \dots, g_m(\alpha)$ , by hypothesis

$$C(\alpha) = \begin{bmatrix} g_1(\alpha) F(\alpha) g_1(\alpha) \dots F^{k_1-1}(\alpha) g_1(\alpha) \dots g_m(\alpha) F(\alpha) g_m(\alpha) \dots F^{k_m-1} g_m(\alpha) \end{bmatrix} \quad (13)$$

is invertible at each  $\alpha \in \Gamma$ . Partitioning  $C^{-1}(\alpha)$  by rows as



$$C^{-1}(\alpha) = \begin{bmatrix} e_{11}(\alpha) \\ \vdots \\ e_{1k_1}(\alpha) \\ \vdots \\ e_{m1}(\alpha) \\ \vdots \\ e_{mk_m}(\alpha) \end{bmatrix} \quad (14)$$

we let

$$Q(\alpha) = \begin{bmatrix} e_{1k_1}(\alpha) \\ e_{1k_1}(\alpha)F(\alpha) \\ \vdots \\ e_{1k_1}(\alpha)F^{k_1-1}(\alpha) \\ \vdots \\ e_{mk_m}(\alpha) \\ e_{mk_m}(\alpha)F(\alpha) \\ \vdots \\ e_{mk_m}(\alpha)F^{k_m-1}(\alpha) \end{bmatrix}, \quad M^{-1}(\alpha) = \begin{bmatrix} e_{1k_1}(\alpha)F^{k_1-1}(\alpha) \\ \vdots \\ e_{mk_m}(\alpha)F^{k_m-1}(\alpha) \end{bmatrix} G(\alpha) \quad (15)$$

and

$$K(\alpha) = -M(\alpha) \begin{bmatrix} e_{1k_1}(\alpha)F^{k_1}(\alpha) \\ \vdots \\ e_{mk_m}(\alpha)F^{k_m}(\alpha) \end{bmatrix} \quad (16)$$

The calculations to verify invertibility of  $Q(\alpha)$  and  $M(\alpha)$  and to show that (10) is satisfied are standard, though tedious unless carried out for a particular case.

Now we need to show that the parameterized linear control law

$$u(t) - u(\alpha) = K(\alpha)[x(t) - x(\alpha)] + M(\alpha)[w(t) - w(\alpha)] \quad (17)$$

with  $w(\alpha) = 0$  is a feedback linearization family, that is, the linearization of some nonlinear

feedback law of the form  $k(x, w)$  about the closed-loop operating point family. But this follows from Corollary 3.1 in [7], since

$$\begin{aligned}
 K(\alpha) \frac{\partial x}{\partial \alpha}(\alpha) + M(\alpha) \frac{\partial w}{\partial \alpha}(\alpha) &= -M(\alpha) \begin{bmatrix} e_{1k_1}(\alpha) F^{k_1}(\alpha) \\ \vdots \\ e_{mk_m}(\alpha) F^{k_m}(\alpha) \end{bmatrix} \frac{\partial x}{\partial \alpha}(\alpha) \\
 &= -M(\alpha) \begin{bmatrix} e_{1k_1}(\alpha) F^{k_1-1}(\alpha) \\ \vdots \\ e_{mk_m}(\alpha) F^{k_m-1}(\alpha) \end{bmatrix} F(\alpha) \frac{\partial x}{\partial \alpha}(\alpha) \\
 &= M(\alpha) \begin{bmatrix} e_{1k_1}(\alpha) F^{k_1-1}(\alpha) \\ \vdots \\ e_{mk_m}(\alpha) F^{k_m-1}(\alpha) \end{bmatrix} G(\alpha) \frac{\partial u}{\partial \alpha}(\alpha) \\
 &= \frac{\partial u}{\partial \alpha}(\alpha)
 \end{aligned} \tag{18}$$

Indeed, an explicit representation for a nonlinear control law corresponding to (17) can be found as follows. Select  $m$  components of  $x(\alpha)$  to form  $\bar{x}(\alpha)$  such that  $\text{rank} [\partial \bar{x} / \partial \alpha](\alpha) = m$ , and denote by  $\bar{x}$  the same selection of  $m$  components from  $x$ . Then a nonlinear feedback law that satisfies (11) is [7]

$$P_{n+1}(x, w) = u(\bar{x}^{-1}(\bar{x})) + K(\bar{x}^{-1}(\bar{x}))[x - x(\bar{x}^{-1}(\bar{x}))] + M(\bar{x}^{-1}(\bar{x}))w \tag{19}$$

Finally, we must consider conditions under which there exists  $z(\alpha)$  such that (9) is satisfied, where  $[\partial P / \partial x](x(\alpha)) = Q(\alpha)$  is specified in (15). It is straightforward to verify that if  $k_i \geq 2$ , then  $e_{ik_i}(\alpha) F^j(\alpha) g_k(\alpha) = 0$  for  $j = 0, 1, \dots, k_i - 2, k = 1, \dots, m$ . Thus, for  $\alpha \in \Gamma$ ,

$$\begin{aligned}
 e_{ik_i}(\alpha) F^{j+1}(\alpha) \frac{\partial x}{\partial \alpha}(\alpha) &= -e_{ik_i}(\alpha) F^j(\alpha) G(\alpha) \frac{\partial u}{\partial \alpha}(\alpha) \\
 &= 0, \quad j = 0, 1, \dots, k_i - 2
 \end{aligned} \tag{20}$$

Therefore (9) is satisfied if  $z_1(\alpha), z_{k_1+1}(\alpha), \dots, z_{k_1 + \dots + k_{m-1} + 1}(\alpha)$  satisfy the following set of  $m$ , uncoupled, total differential equations:

$$\frac{\partial z_1}{\partial \alpha}(\alpha) = e_{1k_1}(\alpha) \frac{\partial x}{\partial \alpha}(\alpha), \quad \frac{\partial z_{k_1+1}}{\partial \alpha}(\alpha) = e_{2k_2}(\alpha) \frac{\partial x}{\partial \alpha}(\alpha), \dots,$$

$$\frac{\partial z_{k_1 + \dots + k_{m-1} + 1}}{\partial \alpha}(\alpha) = e_{mk_m}(\alpha) \frac{\partial x}{\partial \alpha}(\alpha) \quad (21)$$

If  $z(\alpha)$  is such that (9) is satisfied, then a nonlinear variable change that satisfies (12) is [6]

$$P(x) = z(\bar{x}^{-1}(\bar{x})) + Q(\bar{x}^{-1}(\bar{x}))[x - \bar{x}(\bar{x}^{-1}(\bar{x}))] \quad (22)$$

where  $\bar{x}$  and  $\bar{x}^{-1}$  are as in (19).

Writing the  $i^{\text{th}}$  entry of a row vector  $h$  as  $[h]_i$ , and invoking the standard existence condition for solutions to total differential equations [8] we obtain the following sufficient condition for pseudo-linearization.

**Theorem:** Suppose we are given a nonlinear system (1) with linearization family (2) such that (3) is satisfied, and the controllability indices are constant for  $\alpha \in \Gamma$ , where  $\Gamma$  is a sufficiently-small, open neighborhood of  $0 \in R^m$ . Then the nonlinear system is pseudo-linearizable if for all  $\alpha \in \Gamma$ ,

$$\frac{\partial}{\partial \alpha_r} \left[ e_{jk_1}(\alpha) \frac{\partial x}{\partial \alpha}(\alpha) \right]_q = \frac{\partial}{\partial \alpha_q} \left[ e_{jk_1}(\alpha) \frac{\partial x}{\partial \alpha}(\alpha) \right]_r, \quad j = 1, \dots, m, \quad 1 \leq r < q \leq m \quad (23)$$

Furthermore, if these conditions are satisfied, then a pseudo-linearizing transformation is specified by (19) and (22).

For the case  $m = 1$ , it should be clear that (23) is vacuous, and thus pseudo-linearization is always possible under our basic assumptions. Unfortunately, for  $m > 1$  the sufficient condition provided by this theorem is rather restrictive due to the specification of a particular  $K(\alpha)$ ,  $Q(\alpha)$ , and  $M(\alpha)$  out of the many that satisfy (10). This can be relaxed somewhat by introducing integrating factors as follows. In place of (21), we consider

$$\frac{\partial z_1}{\partial \alpha}(\alpha) = r_1(\alpha) e_{1k_1}(\alpha) \frac{\partial x}{\partial \alpha}(\alpha), \dots, \frac{\partial z_{k_1 + \dots + k_{m-1} + 1}}{\partial \alpha}(\alpha) = r_m(\alpha) e_{mk_m}(\alpha) \frac{\partial x}{\partial \alpha}(\alpha) \quad (24)$$

where  $r_1(\alpha), \dots, r_m(\alpha)$  are arbitrary, continuous, real-valued functions that are nonzero for  $\alpha \in \Gamma$ . These functions can be introduced by modifying the definitions of  $Q(\alpha)$  and  $M(\alpha)$  (while leaving  $K(\alpha)$  fixed as in (16)) according to

$$\begin{aligned} \hat{Q}(\alpha) &= \text{block diagonal} \left\{ r_i(\alpha) I_{k_i \times k_i}, i = 1, \dots, m \right\} \cdot Q(\alpha) \\ \hat{M}(\alpha) &= M(\alpha) \cdot \text{diagonal} \left\{ \frac{1}{r_i(\alpha)}, i = 1, \dots, m \right\} \end{aligned} \quad (25)$$

where  $I_{k_i \times k_i}$  is the  $k_i \times k_i$  identity matrix. With  $Q(\alpha)$  and  $M(\alpha)$  replaced by  $\hat{Q}(\alpha)$  and  $\hat{M}(\alpha)$ , (10) remains satisfied and (17) is a feedback linearization family for

$$P_{n+1}(x, w) = u(\bar{x}^{-1}(\bar{x})) + K(\bar{x}^{-1}(\bar{x}))[x - x(\bar{x}^{-1}(\bar{x}))] + \hat{M}(\bar{x}^{-1}(\bar{x}))w \quad (26)$$

where  $\bar{x}$  and  $\bar{x}$  are as before. Also, the equations that determine the nonzero components of  $z(\alpha)$  are precisely those in (24). Therefore, if  $r_1(\alpha), \dots, r_m(\alpha)$  can be found such that the appropriate existence conditions for (24) are satisfied, then the nonlinear variable change

$$P(x) = z(\bar{x}^{-1}(\bar{x})) + \hat{Q}(\bar{x}^{-1}(\bar{x}))[x - x(\bar{x}^{-1}(\bar{x}))] \quad (27)$$

together with (26) accomplish pseudo-linearization. Indeed, for the case  $m = 2$  we will establish that such  $z(\alpha)$  always exists.

**Corollary:** Suppose we are given a nonlinear system (1) with linearization family (2) such that  $m = 1$  or  $m = 2$ , (3) is satisfied, and the controllability indices are constant for  $\alpha \in \Gamma$ , where  $\Gamma$  is a sufficiently-small, open neighborhood of  $0 \in R^m$ . Then the nonlinear system is pseudo-linearizable by a transformation of the form specified in (26) and (27).

**Proof:** For the case  $m = 1$ , we can take  $r_1(\alpha) = 1$ , and (24) becomes

$$\frac{dz_1}{d\alpha}(\alpha) = e_{1n}(\alpha) \frac{dx}{d\alpha}(\alpha) \quad (28)$$

With the initial condition  $z_1(0) = 0$ , we have

$$z_1(\alpha) = \int_0^\alpha e_{1n}(\sigma) \frac{dx}{d\sigma}(\sigma) d\sigma \quad (29)$$

and a pseudo-linearizing transformation can be constructed as in (26) and (27) (or equivalently, (19) and (22)). In the case  $m = 2$ , the conditions for (24) become

$$\frac{\partial}{\partial \alpha_1} \left[ r_j(\alpha) e_{jk_1}(\alpha) \frac{\partial x}{\partial \alpha_2}(\alpha) \right] = \frac{\partial}{\partial \alpha_2} \left[ r_j(\alpha) e_{jk_1}(\alpha) \frac{\partial x}{\partial \alpha_1}(\alpha) \right], \quad j = 1, 2 \quad (30)$$

Performing the indicated differentiations gives two, uncoupled, linear partial differential equations

$$a_j(\alpha) \frac{\partial r_j}{\partial \alpha_1}(\alpha) + b_j(\alpha) \frac{\partial r_j}{\partial \alpha_2}(\alpha) + c_j(\alpha) r_j(\alpha) = 0, \quad j = 1, 2 \quad (31)$$

where

$$\begin{aligned} a_j(\alpha) &= e_{jk_1}(\alpha) \frac{\partial x}{\partial \alpha_2}(\alpha), \quad b_j(\alpha) = -e_{jk_1}(\alpha) \frac{\partial x}{\partial \alpha_1}(\alpha), \\ c_j(\alpha) &= \frac{\partial e_{jk_1}}{\partial \alpha_1}(\alpha) \frac{\partial x}{\partial \alpha_2}(\alpha) - \frac{\partial e_{jk_1}}{\partial \alpha_2}(\alpha) \frac{\partial x}{\partial \alpha_1}(\alpha), \quad j = 1, 2 \end{aligned} \quad (32)$$

Since  $[-b_j(\alpha) \ a_j(\alpha)]$ ,  $j = 1, 2$ , are rows 1 and  $k_1 + 1$  of (9), a simple rank argument shows that  $a_j(\alpha)$  and  $b_j(\alpha)$  do not vanish simultaneously on  $\Gamma$ . Thus (31) can be solved for any initial condition  $r_j(0)$ ,  $j = 1, 2$ . [9] This implies that there exist  $z_1(\alpha)$  and  $z_{k_1+1}(\alpha)$  that satisfy (24), and the proof is complete.

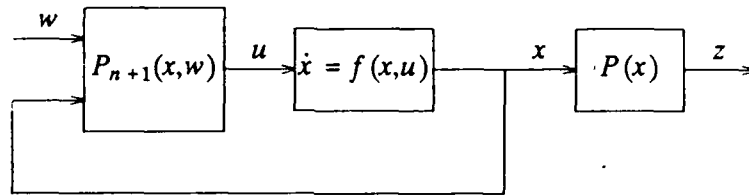
### Conclusions

For systems with one or two inputs, our approach to pseudo-linearization appears to have two main advantages over the original treatment in [1, 2]. These are the simplicity of the derivation based on standard linear theory, and a more explicit representation for a pseudo-linearizing transformation. The major disadvantage of the approach is that it is unclear if the complete results in [2, 3] can be obtained for systems with more than two inputs.

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**Figure 1.** Nonlinear system structure for pseudo-linearization.

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